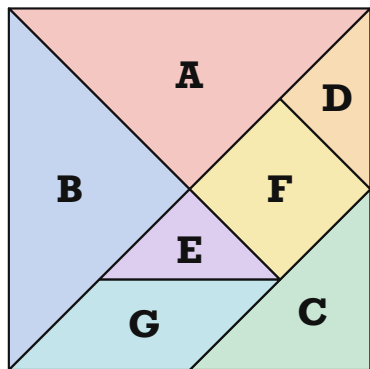


GEOMETRY WORKSHEET · PERIMETER, AREA, CONGRUENCE & SIMILARITY

Tangram Geometry Lab

Seven flat pieces are cut from one 10 cm square. You will measure them, compare them, and prove that no area is lost when the square is taken apart.

**What you will do**

- **Part A:** find the perimeter and area of each piece.
- **Part B:** identify congruent and similar figures.
- **Part C:** find the side, perimeter and area of the whole square.
- **Part D:** show that the seven areas add up to the area of the square.
- **Part E:** investigate more patterns.

You need: a pencil, a ruler, a calculator and scissors. Round answers to 2 decimal places.

A short history of the tangram

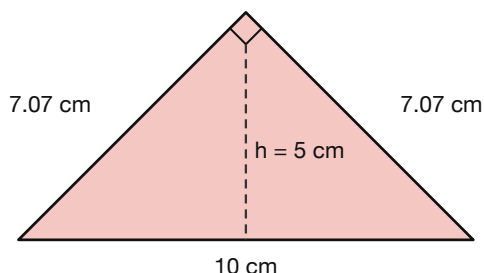
The tangram is a dissection puzzle: a square cut into seven pieces called *tans*. The rules are simple. Use all seven pieces, lay them flat, and do not let them overlap. People have built thousands of silhouettes this way: animals, people, boats, letters and numbers.

Song dynasty	Historians often trace the idea to <i>yanjitu</i> , sets of banquet tables described by the scholar Huang Bosi that could be pushed together into different shapes. A later Ming-dynasty version used triangular "butterfly" tables.
c. 1800	The puzzle appears in China as <i>qīqiǎobǎn</i> , "seven boards of skill." The oldest surviving tangram books were printed around 1813.
1815-1820	Trading ships carried tangram sets to America and Europe. A "tangram craze" swept Europe around 1817-1818, with puzzle books printed in many languages.
Mid-1800s	The English word <i>tangram</i> appears. Nobody is sure where it comes from.
1903	Puzzle maker Sam Loyd published <i>The Eighth Book of Tan</i> , claiming the puzzle was 4,000 years old and invented by a god named Tan. Loyd made the whole story up, but the myth spread for decades.
1942	Mathematicians Fu Traing Wang and Chuan-Chih Hsiung proved that exactly 13 <i>convex</i> shapes (shapes with no dents) can be made from the seven tans.

Every tangram piece is built from the same right isosceles triangle. That makes the tangram a perfect tool for studying area, perimeter, angles and similarity.

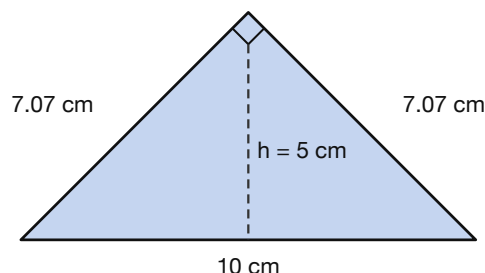
Meet the pieces

All figures are drawn at the same scale (smaller than real size). Lengths are rounded to 2 decimal places. Dashed lines show a height or diagonal. A small square marks a right angle. All angles in the pieces are 45° , 90° or 135° .



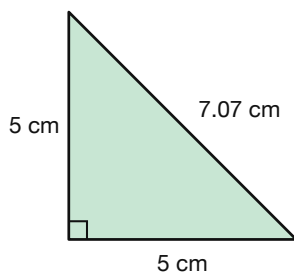
A • Large triangle

Legs 7.07 cm, hypotenuse 10 cm, height to the hypotenuse 5 cm. Angles 45° , 45° , 90° .



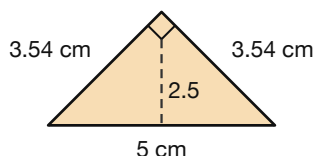
B • Large triangle

Legs 7.07 cm, hypotenuse 10 cm, height to the hypotenuse 5 cm. Angles 45° , 45° , 90° .



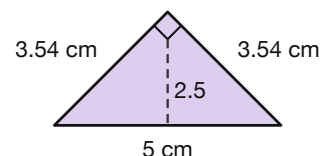
C • Medium triangle

Legs 5 cm, hypotenuse 7.07 cm. Angles 45° , 45° , 90° .



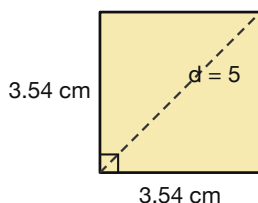
D • Small triangle

Legs 3.54 cm, hypotenuse 5 cm, height to the hypotenuse 2.5 cm.



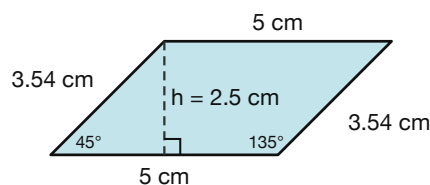
E • Small triangle

Legs 3.54 cm, hypotenuse 5 cm, height to the hypotenuse 2.5 cm.



F • Square

All sides 3.54 cm. Diagonal 5 cm. Four 90° angles.



G • Parallelogram

Long sides 5 cm, short sides 3.54 cm, height 2.5 cm (to the 5 cm base). Angles 45° , 135° , 45° , 135° .

Part A · Perimeter and area of each piece

Perimeter is the distance around a shape: add all its side lengths. Use these area formulas:

Triangle: $A = \frac{1}{2} \times \text{base} \times \text{height}$ (in a right triangle the two legs can be the base and height)

Square: $A = \text{side} \times \text{side}$ Parallelogram: $A = \text{base} \times \text{height}$

PIECE	PERIMETER: SHOW YOUR WORK	AREA: SHOW YOUR WORK
A Large triangle	P = _____ cm	A = _____ cm ²
B Large triangle	P = _____ cm	A = _____ cm ²
C Medium triangle	P = _____ cm	A = _____ cm ²
D Small triangle	P = _____ cm	A = _____ cm ²
E Small triangle	P = _____ cm	A = _____ cm ²
F Square	P = _____ cm	A = _____ cm ²
G Parallelogram	P = _____ cm	A = _____ cm ²

Tip: $3.54 \times 3.54 = 12.53$, not 12.5, because 3.54 is a rounded number. Using the dashed heights on page 2 avoids this rounding problem.

Part B • Congruent and similar figures

Congruent figures have exactly the same shape *and* size: one fits perfectly on top of the other.

Similar figures have the same shape (equal angles, sides in the same ratio) but can be different sizes.

Every pair of congruent figures is also similar.

1. Which pairs of pieces are congruent? Explain how you know.

2. Which pieces are similar to each other? List every group and explain.

3. Find the **scale factor** between the triangles. Divide matching sides (for example, hypotenuse $\div$ hypotenuse).

Small triangle $\rightarrow$ medium triangle: _____

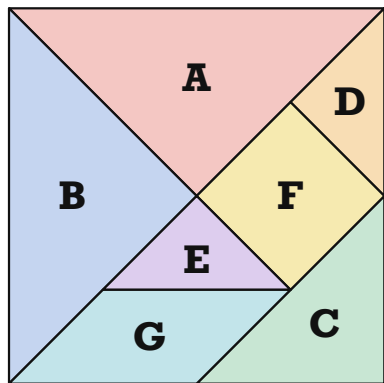
Medium triangle $\rightarrow$ large triangle: _____

Small triangle $\rightarrow$ large triangle: _____

4. Compare the areas of the small, medium and large triangles. When the side lengths are multiplied by a scale factor k , what happens to the area?

5. The medium triangle C, the square F and the parallelogram G all have the same area. Are any of them congruent? Are any of them similar? Explain.

Part C · The completed square



1. Which piece edges make up the top side of the square? Which make up the right side? Use them to find the side length.

Side length = _____ cm

2. Find the perimeter of the square. Show your work.

Perimeter = _____ cm

3. Find the area of the square. Show your work.

Area = _____ cm²

Part D · Do the pieces add up?

When you cut a shape into pieces, no area is created or lost. Copy your areas from Part A, add them, and compare the total with the area of the square from Part C. Then write each piece as a fraction of the whole square.

PIECE	AREA (CM ²)	FRACTION OF THE SQUARE
A · Large triangle		
B · Large triangle		
C · Medium triangle		
D · Small triangle		
E · Small triangle		
F · Square		
G · Parallelogram		
Total		

Complete the statement:

_____ + _____ + _____ + _____ + _____ + _____ + _____ = _____ cm² = area of the square

Does the total equal the area of the square? Explain why this must be true.

Part E · More things to notice

Tick and complete **at least two** of these. Use your cut-out tangram from the last student page.

☐ **Area is kept, perimeter is not**

Add up the perimeters of all seven pieces. Compare the total with the square's perimeter. Why is the total so much bigger?

☐ **The small triangle as a unit**

How many small triangles would cover each piece? How many would cover the whole square?

☐ **Same area, different shapes**

Build a rectangle, a triangle and a parallelogram using all seven pieces. Each has an area of 100 cm^2 . Do they have the same perimeter?

☐ **Perimeter twins**

Two different pieces have the same area *and* the same perimeter. Which ones? Are they congruent?

☐ **Angles in 45° steps**

List every angle in all seven pieces. What do you notice? Why does this help the pieces fit together?

☐ **The piece you must flip**

Which piece cannot match its own mirror image just by sliding or turning? Try it with your cut-out pieces.

☐ **Where $\sqrt{2}$ hides**

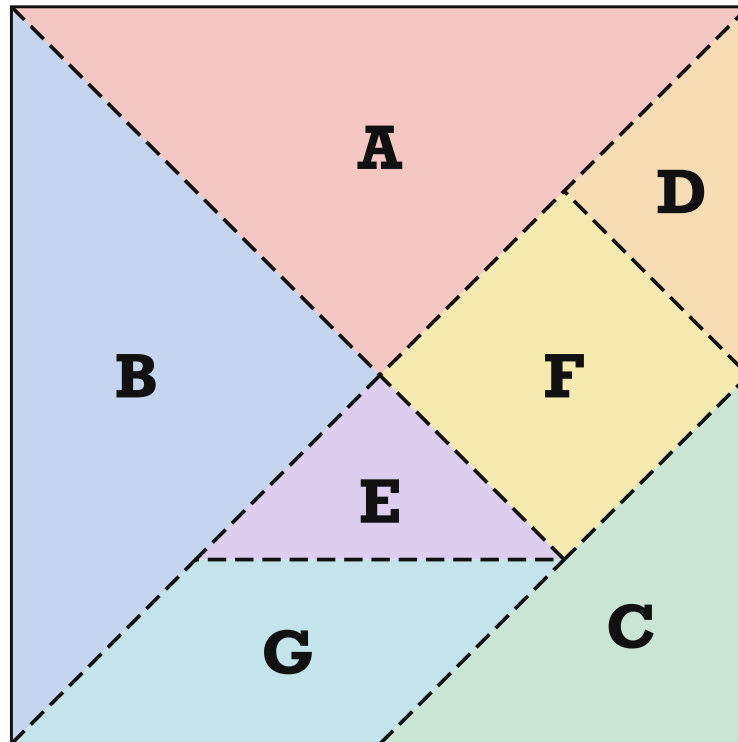
Several lengths are multiples of $\sqrt{2} \approx 1.414$. Could every side of every piece be a whole number of centimetres? Why or why not?

☐ **The 13 convex shapes**

Only 13 convex shapes can be made from all seven pieces. Sketch as many as you can find on the back of this page.

Make your own tangram

Print this page at **Actual size (100%)**, not "Fit to page." Check with a ruler: the outside of the square should measure exactly 10 cm. Then cut along every line to make your seven pieces.



Check your measurements

After cutting, measure the pieces with your ruler. Do your measurements match the numbers on page 2? Record any piece that is off by more than 1 mm and explain why small differences can happen.

Tip: glue this page onto card stock before cutting to make sturdier pieces.

Answer key

Exact values use $\sqrt{2} \approx 1.4142$. Accept decimal answers within about 0.05 of these, since students work from rounded lengths.

Part A • Perimeter and area

PIECE	PERIMETER WORK	PERIMETER	AREA WORK	AREA
A, B • Large	$7.07 + 7.07 + 10$	24.14 cm $10 + 10\sqrt{2}$	$\frac{1}{2} \times 10 \times 5$	25 cm²
C • Medium	$5 + 5 + 7.07$	17.07 cm $10 + 5\sqrt{2}$	$\frac{1}{2} \times 5 \times 5$	12.5 cm²
D, E • Small	$3.54 + 3.54 + 5$	12.07 cm $5 + 5\sqrt{2}$	$\frac{1}{2} \times 5 \times 2.5$	6.25 cm²
F • Square	4×3.54	14.14 cm $10\sqrt{2}$	$(5\sqrt{2}/2)^2 = 50/4$, or $\frac{1}{2} \times 5^2$	12.5 cm²
G • Parallelogram	$5 + 3.54 + 5 + 3.54$	17.07 cm $10 + 5\sqrt{2}$	5×2.5	12.5 cm²

Part B • Congruent and similar figures

1. A and B are congruent (legs 7.07 cm, hypotenuse 10 cm). D and E are congruent (legs 3.54 cm, hypotenuse 5 cm). These are the only congruent pairs.
2. All five triangles (A, B, C, D, E) are similar: each is a right isosceles triangle with angles 45° , 45° , 90° . F and G are not similar to any other piece.
3. Small $\rightarrow$ medium: $7.07 \div 5 \approx \mathbf{1.41}$ ($\sqrt{2}$). Medium $\rightarrow$ large: $10 \div 7.07 \approx \mathbf{1.41}$ ($\sqrt{2}$). Small $\rightarrow$ large: $10 \div 5 = \mathbf{2}$.
4. Areas are 6.25, 12.5 and 25 cm², a ratio of 1 : 2 : 4. The area is multiplied by k^2 . With $k = \sqrt{2}$ the area doubles; with $k = 2$ it is multiplied by 4.
5. No. They have equal areas but different shapes and angles, so they are neither congruent nor similar. Equal area does not mean congruent.

Part C • The completed square

The top side is the hypotenuse of A: **10 cm**. The right side is D's hypotenuse (5 cm) plus one leg of C (5 cm) = 10 cm. Perimeter = $4 \times 10 = \mathbf{40 \text{ cm}}$. Area = $10 \times 10 = \mathbf{100 \text{ cm}^2}$.

Part D • Do the pieces add up?

$25 + 25 + 12.5 + 6.25 + 6.25 + 12.5 + 12.5 = \mathbf{100 \text{ cm}^2}$ = area of the square. Fractions: A and B are $\frac{1}{4}$ each; C, F and G are $\frac{1}{8}$ each; D and E are $\frac{1}{16}$ each. $\frac{1}{4} + \frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{16} + \frac{1}{16} = \frac{16}{16} = 1$. The pieces cover the square exactly once with no gaps or overlaps, so their areas must add to the square's area.

Part E · More things to notice

- **Area is kept, perimeter is not:** total $\approx 2(24.14) + 17.07 + 2(12.07) + 14.14 + 17.07 = \mathbf{120.71 \text{ cm}}$, about three times 40 cm. Inside edges belong to two pieces and get counted twice, but they are not on the outside of the square.
- **The small triangle as a unit:** large = 4, medium = 2, square = 2, parallelogram = 2, small = 1. The whole square is $4 + 4 + 2 + 1 + 1 + 2 + 2 = \mathbf{16}$ small triangles, which matches $100 \div 6.25 = 16$.
- **Same area, different shapes:** no. Shapes with the same area can have different perimeters. Of all rectangles with a given area, the square has the smallest perimeter.
- **Perimeter twins:** the medium triangle C and the parallelogram G (12.5 cm^2 and 17.07 cm each). They are not congruent.
- **Angles in 45° steps:** every angle is 45° , 90° or 135° . Angles meeting at a point must total 360° (or 180° along a straight edge), and multiples of 45° combine easily to make those totals.
- **The piece you must flip:** the parallelogram G. It has no line of symmetry, so its mirror image can only be matched by flipping it over. Every other piece has a line of symmetry.
- **Where $\sqrt{2}$ hides:** no. In a right isosceles triangle the hypotenuse is always $\sqrt{2}$ times a leg, and $\sqrt{2}$ is irrational, so the legs and the hypotenuse can never both be whole numbers.
- **The 13 convex shapes:** 1 triangle, 6 quadrilaterals, 2 pentagons and 4 hexagons (Wang and Hsiung, 1942).

Cut-out page

Small differences (up to about 1 mm) come from printer scaling, cutting accuracy and ruler precision. If every piece is off by the same proportion, the page was probably printed with "Fit to page" turned on.